Proceedings of the ASME 2014 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2014
August 17-20, 2014, Buffalo, New York, USA

DETC2014-34385

STRESS-CONSTRAINED THERMO-ELASTIC TOPOLOGY OPTIMIZATION: A TOPOLOGICAL SENSITIVITY APPROACH

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ABSTRACT*

The focus of this paper is on thermo-elastic topology optimization where the structure is subject to both mechanical and thermal loads. Such problems are of significant importance, for example, in the aircraft industry where structures subject to aerodynamic forces and thermal-gradients must be optimized.

A popular strategy for solving such problems is Solid Isotropic Material with Penalization (SIMP) where pseudo-densities serve as optimization parameters. Yet another strategy is the Rational Approximation of Material Properties (RAMP) that overcomes some of the deficiencies of SIMP. Both methods fundamentally rely on parameterization of the material properties as a function of the pseudo-densities.

Here we consider an alternate level-set approach that relies on the concept of topological sensitivity. The advantages of the proposed method over SIMP and RAMP are: (1) *ad hoc* material parameterization is not required (2) the stresses are well-defined at all points within the evolving topology and (3) the underlying stiffness matrices are always well-conditioned. The proposed method is illustrated through numerical experiments.

1. INTRODUCTION

Topology optimization has rapidly evolved from an academic exercise into an exciting discipline with numerous industrial applications [1], [2]. Applications include optimization of aircraft components [3], [4], spacecraft modules [5], automobiles components [6], cast components [7], compliant mechanisms [8]–[11], etc.

The focus of this paper is on thermo-elastic topology optimization (see Figure 1) where the structure is subject to both mechanical and thermal loads.

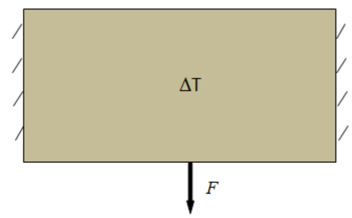


Figure 1: A thermo-elastic problem.

The goal is to find the optimal topology of minimum volume, subject to stress and other constraints. Unlike in pure elastic problems, in thermo-elastic problems, the displacements and stresses are computed after taking into account the additional thermal load. This poses both new theoretical and computational challenges discussed later in the paper.

In Section 2, popular methods for stress-constrained thermo-elastic problems are reviewed. Some of the challenges that remain are identified. In Section 3, we provide a brief review of necessary technical background. Then, in Section 4, the proposed method and its implementation are discussed. In Section 5, numerical experiments are presented, followed by conclusions in Section 6.

2. LITERATURE REVIEW

Current strategies for solving thermo-elastic topology optimization problems can be classified into the following types: homogenization, Solid Isotropic Material with Penalization (SIMP), Rational Approximation of Material Properties (RAMP) and level-set.

Homogenization

In a pioneering work, the authors of [12] adopted the homogenization approach [13] to solve on thermo-elastic topology optimization problems. In particular, they combined asymptotic homogenization on periodic microstructures with thermo-elastic finite element formulations to highlight an important characteristic of such problems, i.e. the final topology of the structure is strongly affected by thermal gradient. From a computational perspective, the increase in check-board patterns for thermo-elastic problems was also observed; methods to overcome these issues were also proposed.

Solid Isotropic Material with Penalization (SIMP)

The most popular approach for topology optimization problems is SIMP. The primary advantage of SIMP is that it is well-understood and relatively easy to implement [14]. Indeed, SIMP has been applied to a variety of topology optimization problems ranging from fluids to non-linear structural mechanics.

In [15], the compliance of a thermo-elastic problem was minimized using SIMP. In [16], the nonlinearity of thermo-elastic topology optimization problem was explored through composites of unusual thermal expansion coefficients. In [17], a design scenario is presented using SIMP for thermo-elastic topology optimization of stiffening thermally restrained thin shell structures.

The 'singularity-problem' associated with zero-density elements in SIMP require careful treatment, for example through epsilon-methods [18], [19]. Secondly, the ill-conditioning of the stiffness matrices, due to low-density elements, can lead to high computational costs for iterative solvers [20], [21]. Challenges include stress-ambiguity and accuracy over gray-elements [22].

RAMP

One of the challenges with the SIMP model is that the material interpolation exhibits zero slope at zero density, posing challenges in thermo-elastic problems [23], [24]. To overcome this deficiency, the Rational Approximation of Material Properties (RAMP) was developed by Stolpe and Svanberg [23]; its superior performance over SIMP was demonstrated in [24]. In [25], a new stress-relaxation method was proposed to include stress constraints, and a group-wise p-norm stress aggregation was adapted for better stress control.

Level-Set

The level-set strategy is gaining popularity for solving topology optimization problems for several reasons: the boundary is well-defined at all times, the stress-singularity problem does not arise, and the stiffness matrices are typically well-conditioned; see [26] for a recent review and comparison of level-set based methods in structural topology optimization.

For purely elastic problems, one of the earliest implementation of level-set based stress-constrained topology optimization appears in [27] where the authors proposed to minimize a domain integral of stress subject to material volume constraint. A topological level-set method for handling stress and displacement constraints in single-load problems was proposed in [28].

For thermo-elastic problems, the authors of [29] adopted a level-set approach to solve a compliance minimization problem, with a volumetric constraint. The primary advantages of the level-set method over SIMP, namely, a well-defined boundary and no intermediate densities, are highlighted and demonstrated.

Proposed

In this paper, we once again adopt the level-set method due to its inherent advantages. However, instead of relying on the Hamilton-Jacobi equations for level-set propagation, the topology sensitivity for thermo-elastic problems is exploited. Thus, the domain need not be initialized with holes. In addition, stress and compliance constraints are considered in the present formulation.

3. TECHNICAL BACKGROUND

3.1 Thermo-elasticity

Finite element formulations of (weakly-coupled) thermo-elastic problems essentially reduce to solving two linear algebra problems:

$$\begin{aligned} K_t t &= q \\ Ku &= f + f^{th} \end{aligned} \tag{1}$$

where:

t: Temperature field

u: Displacement field

K_t: Thermal stiffness matrix

K: Structural stiffness matrix

q: Thermal load

f: Mechanical load

f^{th}: Structural load due to thermal effects

The thermal load vector in Equation (1) is formed via [25]:

$$f_e^{th} = \int_{\Omega_e} B_e^T D_e \varepsilon_e^{th} d\Omega \tag{2}$$

$$\varepsilon_e^{th} = \alpha (t_e - t_0) \Phi^T \tag{3}$$

where:

f_e^{th}: Nodal thermal load vector for each element

Ω_e: Element domain

B_e: Element strain-displacement matrix

D_e: Element elasticity matrix

ε_e^{th}: Element thermal strain vector

α: Thermal expansion coefficient

t_e: Element temperature from thermal analysis

t_0: Reference temperature

Φ: [1 1 1 0 0 0] in 3D; [1 1 0] in 2D

Finally, the stresses are obtained by subtracting the thermal strain from the total strain, and multiplying the resulting strain by the material tensor:

$$\sigma_e = D_e B_e u_e - D_e \varepsilon_e^{th} \tag{4}$$

Further explanations and details may be found, for example, in [30]. The compliance for a thermo-elastic system is defined as:

$$J = (f + f^{th})^T u = u^T K u \tag{5}$$

Observe that Equation (1) represents a weakly-coupled problem where the thermal field influences the displacements, but not the inverse. Strongly-coupled thermo-elastic problems are beyond the scope of this paper.

3.2 PareTO Method

The proposed method for thermo-elastic optimization builds upon the PareTO method for pure elasticity problems described in [21], [31]. The concepts underlying PareTO are therefore summarized next.

PareTO is a topological-sensitivity [32] based method, whose unique feature is that it traces the pareto-optimal curve governing the desired objective φ (such as compliance) and the volume fraction. In other words, the ParetO method is designed to solve the two-objective topology optimization problem:

$$\underset{\Omega \subset D}{Min} \ \{\varphi, |\Omega|\} \\ \text{subject to} \\ \ldots \tag{6}$$

For example, Figure 2 illustrates the pareto-optimal curve and topologies for a 2D compliance minimization problem. The optimization process starts at a volume-fraction of 1 (at the bottom right), and the pareto-optimal curve is traced in small decrements of volume fractions. The optimization terminates if the constraints are violated.

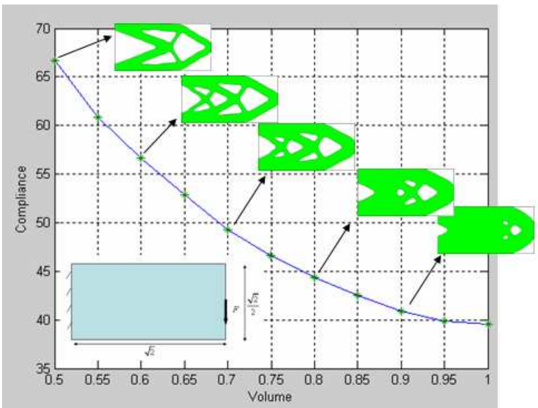


Figure 2: The pareto-optimal curve and topologies.

Topological sensitivity, a central concept in ParetO, captures the first order impact of inserting a small circular hole within a domain on various quantities of interest. This concept has its roots in the influential paper by Eschenauer [33], and has later been extended and explored by numerous authors [32], [34]–[37], including generalization to arbitrary features [38]–[40].

To illustrate, let the quantity of interest be φ (example: compliance). Suppose a tiny hole is introduced as illustrated in Figure 3; the finite element solution u and the quantity φ will change. The topological sensitivity (aka topological derivative) is defined in 2D as:

$$\mathcal{T}_{\varphi}(p) \equiv \lim_{r \to 0} \frac{\varphi(r) - \varphi}{\pi r^2} \tag{7}$$

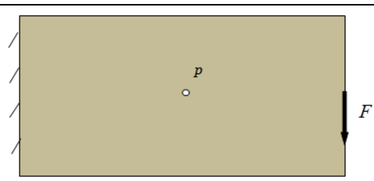


Figure 3: A topological change.

Closed-form expression for the topological sensitivity (TS) can be computed by relying on the classic notions of adjoint (see next Section).

3.3 Topological Level-set

A simple approach to exploiting the TS field is to 'kill' mesh-elements with low values. However, this leads to instability and checker-board patterns. Alternately, the TS field can be used to introduce holes during the topology optimization process via an auxiliary level-set [41]. In ParetO, the topological sensitivity field is used as a level-set, as described next.

For example, consider the compliance TS field illustrated in Figure 4. Given the TS field, and a cutting plane τ, one can define a domain Ω^{τ} per:

$$\Omega^{\tau} = \{p \mid \mathcal{T}_J(p) > \tau\} \tag{8}$$

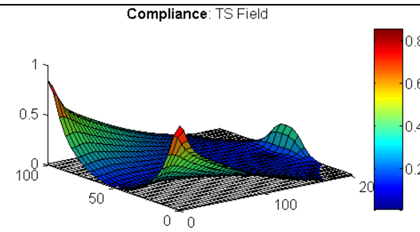


Figure 4: Compliance topological sensitivity (TS) field.

In other words, the domain Ω^{τ} is the set of all points where the topological field exceeds the value of τ; the induced domain Ω^{τ} is illustrated in Figure 5. The τ value can be chosen such that, say, 10% of the volume is removed. Observe how portions of the domain that are least critical for the stiffness of the structure are eliminated.

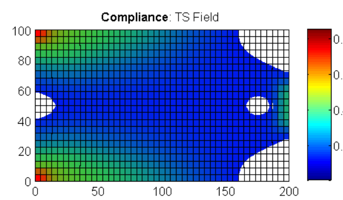


Figure 5: Topological sensitivity field as a level-set.

However, the computed domain may not be 'optimal' [31], i.e., it may not be the stiffest structure for the given volume fraction. One must now repeat the following three steps: (1) solve the finite element problem over Ω^{τ} (2) re-compute the topological sensitivity, and (3) find a new value of τ for the desired volume fraction. In essence, a fixed-point iteration is carried out[37], [42], [21], involving three quantities (see Figure 6): (1) domain Ω^{τ}, (2) displacement fields over Ω^{τ}, and (3) topological sensitivity field over Ω^{τ}.

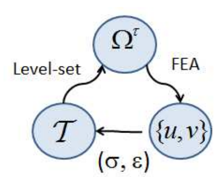


Figure 6: Fixed point iteration involving three quantities

Once convergence has been achieved (in typically 2~3 iterations), an optimal domain at 90% volume fraction will be obtained. An additional 10% volume can now be removed by repeating this process.

4. PROPOSED METHOD

In this paper, we extend the ParetO method to thermo-elastic problems with constraints. In particular, we consider two different formulations described in Section 4.1 and 4.2. In both formulations, a compliance constraint and a stress constraint are imposed as follows:

$$J / J_0 \le \eta_J \quad (9)$$

$$\sigma_{max} / \sigma_0 \le \eta_\sigma \text{ in } \Omega \quad (10)$$

Equation (9) states that the ratio of compliance J of the final topology to the compliance J_0 of the initial topology must not exceed a prescribed value of η_J .

Similarly, Equation (10) states that the ratio of maximum stress σ_{max} (across all elements) in the final topology to the initial maximum stress σ_0 (across all elements) in the initial topology must not exceed a prescribed value of η_σ .

In the numerical experiments, η_J and η_σ range from 1.01 to 10.0, and they control the final termination. On the other hand, the path taken by the optimization process is controlled by choosing one of the two formulations described in Section 4.1 and 4.2.

4.1 Compliance Minimization

In the first formulation the objective φ is the compliance, i.e.:

$$\begin{aligned} &\underset{\Omega\subset D}{Min}\ \{J, |\Omega|\} \\ &J / J_0 \le \eta_J \\ &\sigma_{max} / \sigma_0 \le \eta_\sigma \text{ in } \Omega \\ &\text{subject to} \\ &Ku = f + f^{th} \\ &K_t t = q \end{aligned} \quad (11)$$

In other words, the goal is to trace the pareto-optimal topologies involving compliance and volume fraction until the stress (in any element) exceeds all allowable value, or until the compliance exceeds a specified value. Such topologies will be referred to as *stiff topologies* for thermo-elastic problems.

4.2 Stress-Minimization

In the second formulation φ is the p-norm stress measure, i.e.:

$$\begin{aligned} &\underset{\Omega\subset D}{Min}\ \{\sigma_p, |\Omega|\} \\ &J / J_0 \le \eta_J \\ &\sigma_{max} / \sigma_0 \le \eta_\sigma \text{ in } \Omega \\ &\text{subject to} \\ &Ku = f + f^{th} \\ &K_t t = q \end{aligned} \quad (12)$$

where σ_p is the p-norm stress measure [43] of the von Mises stress over all elements:

$$\sigma_p = \left(\sum_e (\sigma_e)^p \right)^{1/p} \quad (13)$$

The goal is to trace the pareto-optimal topologies involving the global stress measure and volume fraction until the stress (in any element) exceeds a specified value, or until the compliance exceeds a specified value. Such topologies will be referred to here as *strong* topologies.

Generating 'strong' topologies is computationally more expensive than generating 'stiff' topologies [44], but arguably more important [45].

4.3 Topological Sensitivity Fields

For each of the two formulations, the topological sensitivity field associated with the objective φ must be computed. For the compliance, the topological sensitivity expression is well-known and is given by [46]:

$$\mathcal{T}_J = \frac{4}{1+\nu}\sigma : \varepsilon - \frac{1-3\nu}{1-\nu^2} tr(\sigma) tr(\varepsilon) \quad (14)$$

Note that the strain fields in Equation (14) is the total strain, while the stress field is computed via Equation (4).

For the p-norm stress field, the topological sensitivity depends not only on the primary displacement field u but also on an adjoint field λ [28]:

$$\mathcal{T}_{\sigma_p} = \frac{4}{1+\nu}\sigma(u) : \varepsilon(\lambda) - \frac{1-3\nu}{1-\nu^2} tr(\sigma(u)) tr(\varepsilon(\lambda)) \quad (15)$$

The adjoint field associated with the p-norm stress, by definition, satisfies the following equation [47]:

$$K\lambda = -\nabla_u \left(\sigma_p\right) \quad (16)$$

Using the definition in Equation (13), one can show that (see [28]):

$$\nabla_u \left(\sigma_p\right) = -\frac{1}{p}\left(\sum_e \left(\sigma_e\right)^p\right)^{1/p-1} \left[\sum_e g_e\right] \quad (17)$$

and

$$\begin{aligned} &g_e = 0.5p\left(\sigma_e\right)^{p-2} \begin{pmatrix} \left(2\sigma_{11} - \sigma_{22}\right)\left(F_{1,:}\right) + \left(2\sigma_{22} - \sigma_{11}\right)\left(F_{2,:}\right) + \\ 6\sigma_{12} F_{3,:} \end{pmatrix} \\ &[F] = [D][B] \end{aligned} \quad (18)$$

4.4 Algorithm

Once the topological sensitivities can be computed, the overall algorithm (for both formulations) is fairly simple, and proceeds as follows:

1. The optimization starts at a volume fraction of 1.0. The 'current volume fraction' v is set to 1.0, and 'volume decrement' Δv , is set to 0.05.
2. The thermal and structural finite element problems in Equation (1) are solved, and the total strain and stress are extracted at the center of each element. For the stress-objective, an additional adjoint problem in Equation (16) is solved.
3. The topological sensitivity field (Equation (14) or (15)) is computed at the center of each element, and locally smoothened with neighboring elements.
4. Treating the topological sensitivity field as a level-set, a new topology with a volume fraction of ($v - \Delta v$) is extracted. The compliance is computed over the new topology. If the compliance has converged, then the optimization moves to the next step, else it returns to step 2.

5. The current volume fraction is set to ($v - \Delta v$), and the optimization returns to step 2, until the final volume fraction is reached or until one of the constraints is violated.

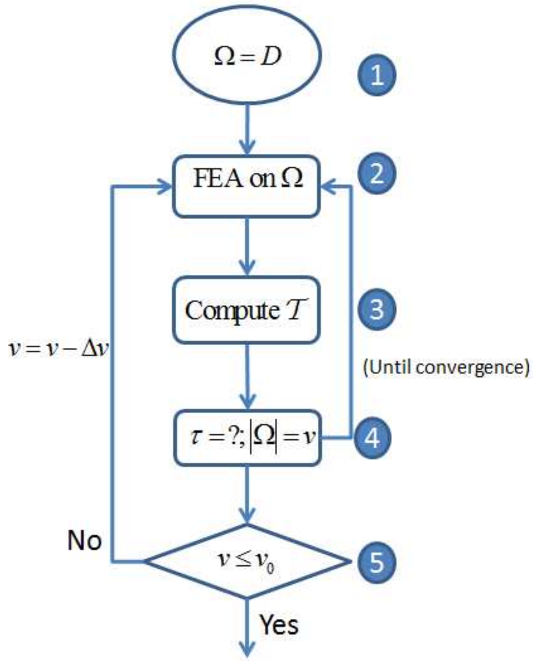


Figure 7: An overview of the algorithm.

5. NUMERICAL EXPERIMENTS

In this Section, we demonstrate the proposed method through numerical experiments. The default parameters are as follows:

- The material is assumed to be steel, i.e., the elastic modulus is $E = 2e11\,Pa$, the Poisson's ratio is $\nu = 0.3$ and the coefficient of thermal expansion $\alpha = 1.1e-5$.
- The reference temperature is zero C, and a thermal load is applied by increasing the temperature uniformly by ΔT
- Unless otherwise noted, the p-norm value is 8.
- Bilinear quad elements are used for finite element analysis.

For all experiments, the constraints are:

$$J / J_0 \le 3.0 \quad (19)$$

$$\sigma_{\max} / \sigma_0 \le 1.5 \quad (20)$$

Further, the desired volume fraction is 0.1. In other words, the optimization terminates if the constraints are violated or if the final volume fraction is reached.

5.1 Bi-clamped beam with a point load

The first experiment is inspired by the classic bi-clamped structure which was previously studied by Rodrigues and Fernandes [12]. As illustrated in Figure 8, the structure is clamped on right and left edges and a mechanical point load of $F = 1e6N$ is applied at the center of the bottom edge; the structure is also subject to a homogeneous temperature increase of ΔT . The domain is meshed with 4500 elements.

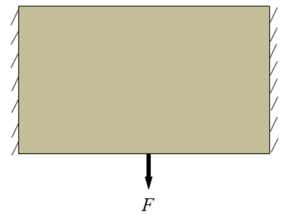
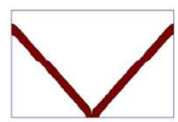
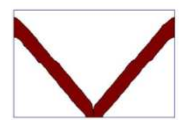
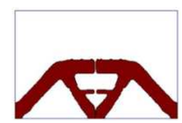


Figure 8: The bi-clamped structure with point-load.

Compliance Formulation (Stiff Designs)

The results of the *compliance minimization* problem for three different levels of temperature loadings are summarized in Table 1. Observe that the final volume fraction and the topologies are a strong function of the temperature increase. Compared to the default case (middle column), a lower volume fraction is reached with decrease in temperature (left column); both are compliance constrained. On the other hand, with an increase in temperature, the structure is stress constrained (stress constraints are difficult to meet exactly).

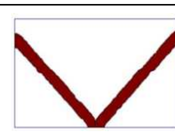
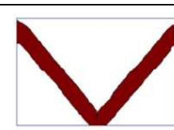
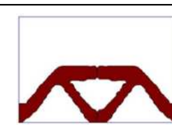
Table 1: Final topologies and results for compliance minimization of the bi-clamped structure.

ΔT	-1.0	0.0	1.0
Final topolog y			
v_{final}	0.106	0.175	0.205
J_{final}	$2.89J_0$	$2.98J_0$	$2.16J_0$
σ_{final}	$0.99\sigma_0$	$1.00\sigma_0$	$1.37\sigma_0$

Stress Formulation (Strong Designs)

The results of the *stress* minimization problem are summarized in Table 2. The results are similar to that of Table 1, except that in the last column, a lower volume fraction has been reached due to the compliance constraint. This highlights the difference between tracing compliance-minimization and tracing stress-minimization

Table 2: Final topologies and results for stress minimization of the bi-clamped structure.

ΔT	-1.0	0.0	1.0
Final topolog y			
v_{final}	0.112	0.183	0.155
J_{final}	$2.96J_0$	$2.97J_0$	$3.00J_0$
σ_{final}	$1.08\sigma_0$	$1.00\sigma_0$	$1.20\sigma_0$

5.2 Bi-clamped beam with distributed loads

Next we consider a similar bi-clamped beam but with distributed loads on the top edge as shown in Figure 9 [12]. The dimension of this beam is $0.5m \times 0.28m \times 0.01m$ and the distributed load is $6e9N/m^2$. The domain is meshed with 3200 finite elements and subject to three different uniform temperature changes.

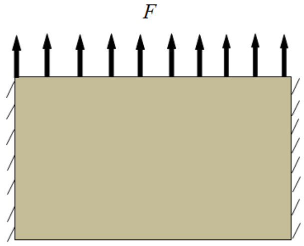
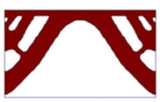
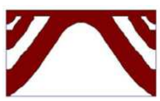


Figure 9: The bi-clamped structure with a distributed load.

Compliance Formulation (Stiff Designs)

The results of the *compliance minimization* problem for three different levels of temperature loadings are summarized in Table 3. Some minor differences in topologies are noted.

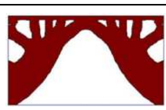
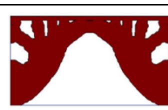
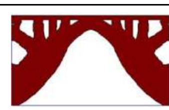
Table 3: Final topology and results for compliance minimization of the bi-clamped structure with distributed load.

ΔT	-20	0.0	20
Final topology			
v_{final}	0.355	0.40	0.375
J_{final}	$2.95J_0$	$2.88J_0$	$2.93J_0$
σ_{final}	$1.47\sigma_0$	$1.47\sigma_0$	$1.45\sigma_0$

Stress Formulation (Strong Designs)

The results of the corresponding *stress* minimization problem are summarized in Table 4. The topologies are consistent with those reported in [12]; relaxing the compliance constraint would result in a lower volume fraction.

Table 4: Final topologies and results for stress-minimization of the bi-clamped structure with distributed loads.

ΔT	-20	0.0	20
Final topology			
v_{final}	0.478	0.488	0.468
J_{final}	$2.99J_0$	$3.00J_0$	$2.99J_0$
σ_{final}	$1.43\sigma_0$	$1.41\sigma_0$	$1.42\sigma_0$

5.3 Clamped beam with tip load

The next example is illustrated in Figure 10, where a beam that is 1.5m long, 1m wide and 0.01m thick, is clamped on the left edge and subject to a point load $F = 5e8N$. The geometry is meshed with 3000 elements.

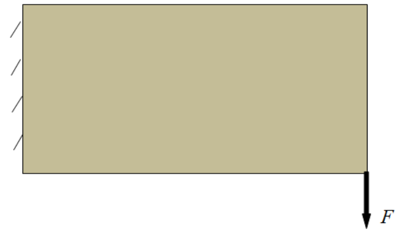
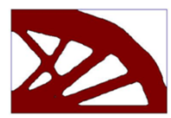
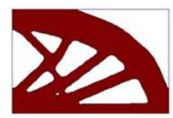


Figure 10: The cantilever beam problem.

Compliance Formulation (Stiff Designs)

The results of the compliance minimization problem for three different levels of temperature loadings are summarized in Table 5. The impact of temperature on the final result is minimal in this case, i.e., the structure is largely dominated by the external force.

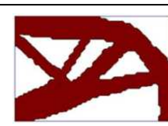
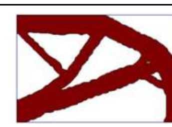
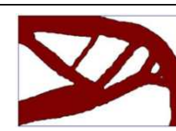
Table 5: Final topologies and results for compliance minimization of cantilever beam.

ΔT	0	5	10
Final topology			
v_{final}	0.55	0.56	0.53
J_{final}	$1.55J_0$	$1.52J_0$	$1.60J_0$
σ_{final}	$1.50\sigma_0$	$1.49\sigma_0$	$1.50\sigma_0$

Stress Formulation (Strong Designs)

The results of the stress minimization problem are summarized in Table 6. The topologies are significantly different from those in Table 5. Also note that for a temperature increase of 5, the final stress is much closer to the constraint of 1.5, hence a lower volume fraction has been reached; this is purely a numerical artifact.

Table 6: Final topologies and results for stress minimization of cantilever beam.

ΔT	0	5	10
Final topology			
v_{final}	0.42	0.38	0.44
J_{final}	2.47_0	$2.85J_0$	$2.55J_0$
σ_{final}	$1.43\sigma_0$	$1.49\sigma_0$	$1.48\sigma_0$

5. CONCLUSIONS

The main contribution of the paper is a new method for stress constrained topology optimization of thermo-elastic problems. Two different formulations were presented and compared. Both formulations exploit the concept of topological sensitivity; thus material parameterization is not required.

As the numerical experiments reveal, the impact of small temperature variations on the final topologies can be significant for certain problems, and minimal for other problems. Future work will focus on including other constraints including buckling and eigen-modes.

PareTO can be downloaded from www.ersl.wisc.edu

Acknowledgements

The authors would like to thank the support of National Science Foundation through grants CMMI-1232508 and CMMI-1161474, and the support of the Wright-Patterson Air-force Laboratory.

REFERENCES

[1] Eschenauer H. A. and Olhoff N., "Topology optimization of continuum structures: A review," *Applied Mechanics Review*, vol. 54, no. 4, pp. 331–389, 2001.

[2] Rozvany G. I. N., "A critical review of established methods of structural topology optimization," *Structural and Multidisciplinary Optimization*, vol. 37, no. 3, pp. 217–237, 2009.

[3] Kesseler E., "Multidisciplinary design analysis and multi-objective optimisation applied to aircraft wing," *WSEAS transactions on systems and Control and Cybernetics*, vol. 1, no. 2, p. 221 227, 2006.

[4] Alonso J. J., "Aircraft design optimization," *Mathematics and Computers in Simulation*, vol. 79, no. 6, pp. 1948–1958, 2009.

[5] Coverstone V. H., "Optimal multi-objective low-thrust spacecraft trajectories," *Comput. Methods Appl. Mech. Eng.*, vol. 186, pp. 387–402, 2000.

[6] Wang L., "Automobile body reinforcement by finite element optimization," *Finite Elements in Analysis and Design*, vol. 40, no. 8, pp. 879 – 893, 2004.

[7] Harzheim L., "A review of optimization of cast parts using topology optimization II-Topology optimization with manufacturing constraints," *Structural and Multidisciplinary Optimization*, vol. 31,, no. 5, pp. 388–299, 2006.

[8] Ananthasuresh G. K., S. Kota, and Y. Gianchandani, "A methodical approach to the design of compliant micromechanisms," in *Solid State Sensor and Actuator Workshop*, 1994, pp. 189–192.

[9] Nishiwaki S., "Topology Optimization of Compliant Mechanisms using the Homogenization Method," *International Journal for Numerical Methods in Engineering*, vol. 42, pp. 535–559, 1998.

[10] Bruns T. E. and Tortorelli D. A., "Topology optimization of non-linear elastic structures and compliant mechanisms," *Computer Methods in Applied Mechanics and Engineering*, vol. 190, no. 26–27, pp. 3443–3459, 2001.

[11] Luo Z., "Compliant mechanism design using multi-objective topology optimization scheme of continuum structures," *Structural and Multidisciplinary Optimization*, vol. 30, pp. 142–154, 2005.

[12] Rodrigues H. and Fernades H., "A material based model for topology optimization of thermoelastic structures," *International Journal for Numerical Methods in Engineering*, vol. 38, pp. 1951–65, 1995.

[13] Bendsøe M. P. and Kikuchi N., "Generating optimal topologies in structural design using a homogenization method," *Computer Methods in Applied Mechanics and Engineering*, vol. 71, pp. 197–224, 1988.

[14] Sigmund O., "A 99 line topology optimization code written in Matlab," *Structural and Multidisciplinary Optimization*, vol. 21, no. 2, pp. 120–127, 2001.

[15] Li D. and Zhang X., "Topology Optimization of Thermo-Mechanical Continuum Structure," presented at the 2010 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Canada, 2010.

[16] Sigmund O. and Torquato S., "Design of materials with extreme thermal expansion using a three-phase topology optimization method," *Journal of the Mechanics and Physics of Solid*, vol. 45, pp. 1037–1067, 1997.

[17] Deaton J. and Grandhi R. V., "Stiffening of Thermally Restrained Structures via Thermoelastic Topology Optimization," presented at the 53rd AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics and Materials Conference, Honolulu, Hawaii, 2012.

[18] Guo X. and Cheng G. D., "Epsilon-continuation approach for truss topology optimization," *Acta Mechanica Sinica*, vol. 20, no. 5, pp. 526–533, 2004.

[19]Le C., "Developments in topology and shape optimization," PhD thesis, University of Illinois at Urbana-Champaign, Urbana-Champaign, 2010.

[20] Wang S., Sturler E. D., and Paulino G., "Large-scale topology optimization using preconditioned Krylov subspace methods with recycling," *International Journal for Numerical Methods in Engineering*, vol. 69, no. 12, pp. 2441–2468, 2007.

[21] Suresh K., "Efficient Generation of Large-Scale Pareto-Optimal Topologies**," *Structural and Multidisciplinary Optimization*, vol. 47, no. 1, pp. 49–61, 2013.

[22] Guo X., Zhang W. S., Wang Y. U., and Wei P., "Stress-related topology optimization via level set approach," *Comput. Methods Appl. Mech. Eng.*, vol. 200, pp. 3439–3452, 2011.

[23] Stolpe M. and Svanberg K., "An alternative interpolation scheme for minimum compliance topology optimization," *Structural and Multidisciplinary Optimization*, vol. 22, pp. 116–124, 2001.

[24] Gao T. and Zhang W., "Topology optimization involving thermo-elastic stress loads," *Structural and Multidisciplinary Optimization*, vol. 42, pp. 725–738, 2010.

[25] Deaton J. and Grandhi R. V., "Topology Optimization of Thermal Structures with Stress Constraints," presented at the 54th AIAA/ASME/ASCE/ASC Structures, Structural Dynamics, and Materials Conference, Boston, MA, 2013.

[26]van Dijk N. P., Maute K., Langelaar M., and van Keulen F., "Level-set methods for structural topology optimization: a review," *Structural and Multidisciplinary Optimization*, vol. DOI 10.1007/s00158–013–0912-y, 2013.

[27] Allaire G. and Jouve F., "Minimum stress optimal design with the level set method," *Engineering Analysis with Boundary Elements*, vol. 32, no. 11, pp. 909–918, 2008.

[28] Suresh K. and Takalloozadeh M., "Stress-Constrained Topology Optimization: A Topological Level-Set

Approach," *Structural and Multidisciplinary Optimization*, vol. Submitted, 2012.
[29] Xia Q. and Wang M. Y., "Topology Optimization of Thermoelastic Structures Using Level Set Method," presented at the EngOpt 2008 - International Conference on Engineering Optimization, Rio de Janeiro, Brazil, 2008.
[30] Hetnarski R. B., Ignaczak J., Noda N., Sumi N., and Tanigawa Y., *Theory of Elasticity and Thermal Stresses: Explanations, Problems and Solutions*. Springer, 2013.
[31] Suresh K., "A 199-line Matlab code for Pareto-optimal tracing in topology optimization," *Structural and Multidisciplinary Optimization*, vol. 42, no. 5, pp. 665–679, 2010.
[32] Sokolowski J. and Zochowski A., "On Topological Derivative in Shape Optimization," *SIAM journal on control and optimization*, vol. 37, no. 4, pp. 1251–1272, 1999.
[33] Eschenauer H. A., Kobelev V. V., and Schumacher A., "Bubble method for topology and shape optimization of structures," *Structural Optimization*, vol. 8, pp. 42–51, 1994.
[34] Novotny A. A., Feijóo R. A., Padra C., and Taroco E., "Topological Derivative for Linear Elastic Plate Bending Problems," *Control and Cybernetics*, vol. 34, no. 1, pp. 339–361, 2005.
[35] Novotny A. A., "Topological-Shape Sensitivity Method: Theory and Applications," *Solid Mechanics and its Applications*, vol. 137, pp. 469–478, 2006.
[36] Novotny A. A., Feijoo R. A., and Taroco E., "Topological Sensitivity Analysis for Three-dimensional Linear Elasticity Problem," *Computer Methods in Applied Mechanics and Engineering*, vol. 196, no. 41–44, pp. 4354–4364, 2007.
[37] Céa J., Garreau S., Guillaume P., and Masmoudi M., "The shape and topological optimization connection," *Computer Methods in Applied Mechanics and Engineering*, vol. 188, no. 4, pp. 713–726, 2000.
[38] Turevsky I., Gopalakrishnan S. H., and Suresh K., "An Efficient Numerical Method for Computing the Topological Sensitivity of Arbitrary Shaped Features in Plate Bending**," *International Journal of Numerical Methods in Engineering*, vol. 79, pp. 1683–1702, 2009.
[39] Turevsky I. and Suresh K., "Generalization of Topological Sensitivity and its Application to Defeaturing**," in *ASME IDETC Conference*, Las Vegas, 2007.
[40] Gopalakrishnan S. H. and Suresh K., "Feature Sensitivity: A Generalization of Topological Sensitivity**," *Finite Elements in Analysis and Design*, vol. 44, no. 11, pp. 696–704, 2008.
[41] Allaire G., Jouve F., and Toader A. M., "Structural Optimization using Sensitivity Analysis and a Level-set Method," *Journal of Computational Physics*, vol. 194, no. 1, pp. 363–393, 2004.
[42] Norato J. A., Bendsøe M. P., Haber R. B., and Tortorelli D. A., "A topological derivative method for topology optimization," *Structural and Multidisciplinary Optimization*, vol. 33, pp. 375–386, 2007.
[43] Le C., Norato J. A., Bruns T. E., Ha C., and Tortorelli D. A., "Stress-based topology optimization for continua," *Structural and Multidisciplinary Optimization*, vol. 41, no. 4, pp. 605–620, 2010.
[44] Yang R. J. and Chen C. J., "Stress-Based Topology Optimization," *Structural Optimization*, vol. 12, pp. 98–105, 1996.
[45] Duysinx P., Miegroet L. V., Lemaire E., Brüls O., and Bruyneel M., "Topology and generalized shape optimization: Why stress constraints are so important?," *Int. J. Simul. Multidisci. Des. Optim.*, vol. 2, pp. 253–258, 2008.
[46] Turevsky I. and Suresh K., "Tracing the Envelope of the Objective-Space in Multi-Objective Topology Optimization**," presented at the ASME IDETC/CIE Conference, Washington, DC, 2011.
[47] Choi K. K. and Kim N. H., *Structural Sensitivity Analysis and Optimization I: Linear Systems*. New York: Springer, 2005.